

**14.01 Fall 2010**  
**Problem Set 2 Solutions**

1. (10 points) Annie and David are painting their apartment. At the paint store, David says he prefers Canary Yellow to Bumblebee Yellow, Lime Yellow, and Crayola Yellow. Annie finds new paint samples and asks David to compare Canary Yellow to School Bus Yellow and to Sunrise Yellow. David prefers Sunrise Yellow to Canary Yellow, and prefers School Bus Yellow to Canary Yellow. He also prefers Sunrise Yellow to School Bus Yellow. The store is out of Sunrise Yellow, so they buy School Bus Yellow and paint their apartment with it. David then insists that they go back, buy Lime Yellow, and repaint the apartment.

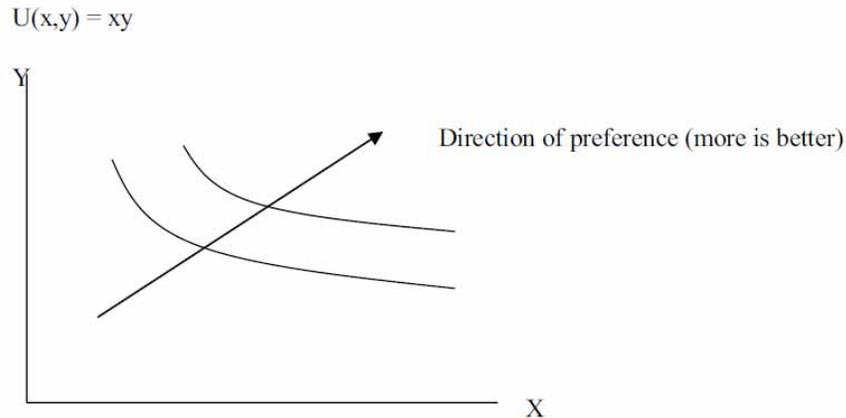
True/False/Uncertain: David has rational preferences (as we define them).

False. He violates transitivity.

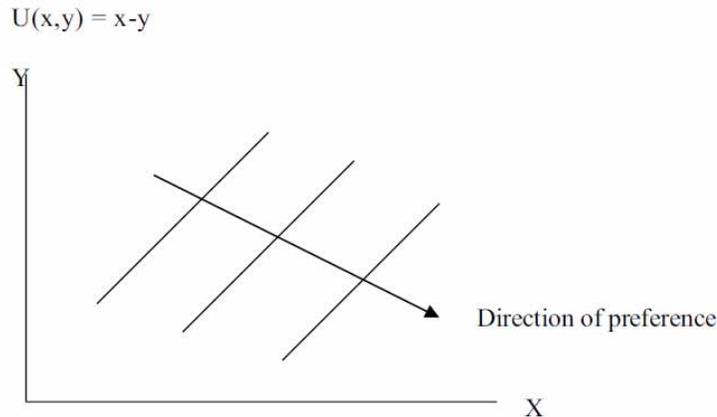
Problem 1 solution courtesy of William Wheaton. Used with permission.

2. (20 points) In each of the following examples, a consumer purchases just two goods:  $x$  and  $y$ . Based on the information in each of the following parts, sketch a plausible set of indifference curves (that is, draw at least two curves on a set of labeled axes, and indicate the direction of higher utility). Also, write down a utility function  $u(x, y)$  consistent with your graph. Note that although all these preferences should be assumed to be complete and transitive (as required for utility representation), not all will be monotone.

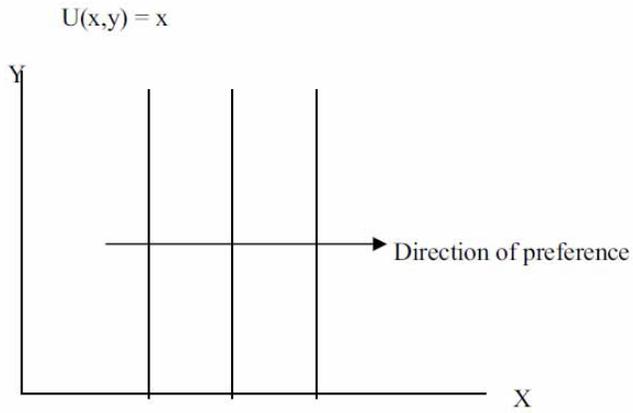
- (a) (4 points) Jessica enjoys bagels  $x$  and coffee  $y$ , and consuming more of one makes consuming the other more enjoyable.



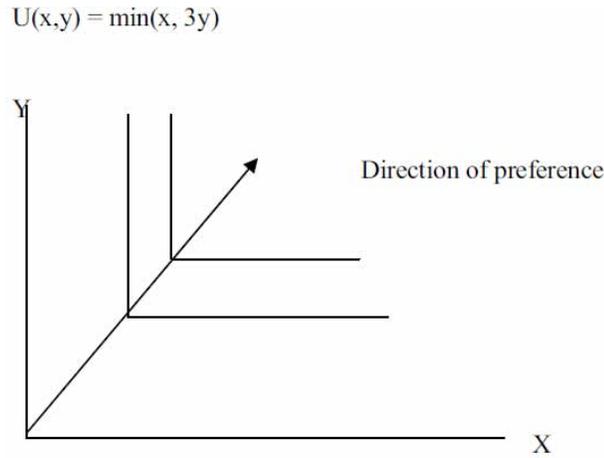
- (b) (4 points) Plamen loves mocha swirl ice cream  $x$ , but he hates mushrooms  $y$ .



(c) (4 points) Jennifer likes Cheerios  $x$ , and neither likes nor dislikes Frosted Flakes  $y$ .

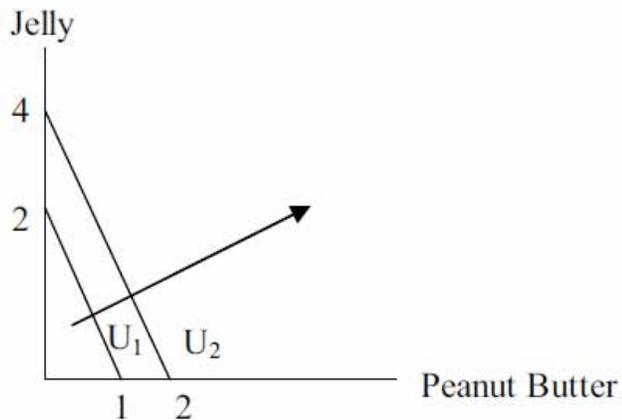


(d) (4 points) Edward always buys three white tank tops  $x$  for every pair of jeans  $y$ .



(e) (4 points) Nancy likes both peanut butter  $x$  and jelly  $y$ , and always gets the same additional satisfaction from an ounce of peanut butter as she does from two ounces of jelly.

$U(x,y) = 2x+y$  where  $U_1 < U_2$



3. (20 points) A consumer's preferences are representable by the following utility function:

$$u(x, y) = x^{\frac{1}{2}} + y.$$

(a) (10 points) Obtain the MRS of the consumer at an arbitrary point  $(\bar{x}, \bar{y})$ , where  $\bar{x} > 0$  and  $\bar{y} > 0$ .

$$MRS = -\frac{\frac{1}{2}x^{-\frac{1}{2}}}{1} = -\frac{1}{2}x^{-\frac{1}{2}}$$

(b) (10 points) Suppose the price of the second good ( $y$ ) is 1, and the price of the first good ( $x$ ) is denoted by  $p > 0$ . If the consumer's income is  $m > 0$ , obtain the optimal consumption bundle of the consumer (in terms of  $m$  and  $p$ ). [Caution: make sure you cover cases in which  $m$  is relatively low, as well as cases in which  $m$  is relatively high.]

The consumer solves  $\max x^{\frac{1}{2}} + y$  so that  $px + y = m$ . We look for stationary values of the Lagrangian  $L = x^{\frac{1}{2}} + y + \lambda(m - px - y)$ . The first-order conditions for stationarity are

$$\begin{aligned} \frac{\partial L}{\partial x} &= \frac{1}{2}x^{-\frac{1}{2}} - \lambda p = 0 \\ \frac{\partial L}{\partial y} &= 1 - \lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= m - px - y = 0 \end{aligned}$$

Combining the first two equations above gives  $\frac{1}{2x^{\frac{1}{2}}} = p$ , or  $x^* = \frac{1}{4p^2}$ . Substituting  $x^*$  into the budget constraint gives  $y = m - px^* = m - \frac{1}{4p}$ .

Case 1)  $m \geq \frac{1}{4p} \rightarrow x^* = \frac{1}{4p^2}$  and  $y = m - \frac{1}{4p} \geq 0$ .

Case 2)  $m \leq \frac{1}{4p} \rightarrow x^* = \frac{m}{p}$  and  $y = 0$ .

Graphically,

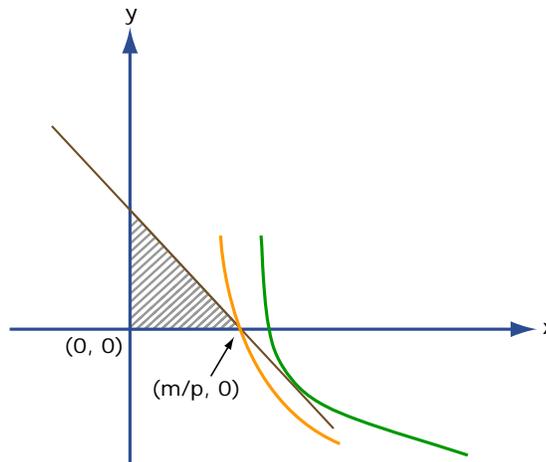


Image by MIT OpenCourseWare.

4. (25 points) It is exactly 24 hours before Lauren's physics final. She has an economics final directly after the physics final and has no time to study in between. Lauren wants to be a physicist, so she places more weight on her physics test score. Her utility function is given by

$$u(p, e) = 0.6 \ln(p) + 0.4 \ln(e)$$

where  $p$  is the score on the physics final and  $e$  is the score on the economics final. Although she cares more about physics, she is better at economics; for each hour spent studying economics she will increase her score by 3 points, but her physics score will only increase by 2 points for every hour spent studying physics. Studying zero hours results in a score of zero on both subjects (although  $\ln(0)$  is not defined, assume her utility for a score of zero is negative infinity).

- (a) (5 points) What constraints does Lauren face in her test score maximization problem?

$$\begin{aligned}H_p + H_e &\leq 24 \\ H_p &\geq 0, \quad H_e \geq 0\end{aligned}$$

One might also consider production constraints:

$$p = 2H_p, \quad e = 3H_e$$

- (b) (5 points) How many hours should Lauren optimally spend studying physics? How many hours studying economics? (hours are divisible)

$$\begin{aligned}\max_{H_e} & 0.6 \ln(2 \times (24 - H_e)) + 0.4 \ln(3H_e) \\ \rightarrow \text{F.O.C.} & 0.6 \times \frac{2}{48 - 2H_e} = 0.3 \times \frac{1}{H_e} \\ \rightarrow & H_e = 9.6, \quad H_p = 14.4\end{aligned}$$

- (c) (5 points) What economics and physics test scores will she achieve (i.e. what are  $e^*$  and  $p^*$ )?

$$\begin{aligned}e^* &= 3H_e = 28.8 \\ p^* &= 2H_p = 28.8\end{aligned}$$

- (d) (5 points) What utility level will she achieve?

$$u(p, e) = 0.6 \ln(p) + 0.4 \ln(e) = 3.36$$

- (e) (5 points) Suppose Lauren can get an economics tutor. If she goes to the tutor, she will increase her economics test score by 5 points for every hour spent studying instead of 3 points, but will lose 4 hours of study time by going to the tutor. She cannot study while at the tutor, and going to the tutor does not directly improve her test score. Should Lauren go to the tutor?

$$\begin{aligned}\max_{H_e} & 0.6 \ln(2 \times (20 - H_e)) + 0.4 \ln(5 \times H_e) \\ \text{F.O.C.} & 0.6 \times \frac{2}{40 - 2H_e} = \frac{0.4}{H_e} \\ \rightarrow & H_e = 8, \quad H_p = 20 - H_e = 12 \\ \rightarrow & e = 40, \quad p = 24, \quad u(24, 40) = 3.38\end{aligned}$$

Lauren has higher utility when she goes to the tutor, so she should go.

Problem 4 solution courtesy of William Wheaton. Used with permission.

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